higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

Tshwane South TVET College
"achieve the future"

SUBJECT: Foundational Mathematics

LEVEL : PLP

MODULE : 3.

Unit NO : Unit 6

UNIT NAME: PROPERTIES OF A TRIANGLE

CONTENT

After completing this topic, you will be able to:

- Identify different types of triangle and angles on triangle
- The calculate exterior angle
- Do calculations of unknown using mathematics tools
- Pythagoras's Theorem

UNIT 6: PROPERTIES OF TRIANGLES

1. When you have completed this unit you will be able to:
    a. Identify an equilateral triangle
    b. Identify an isosceles triangle
    c. Identify a scalene triangle
    d. Identify an acute angled triangle
    e. Identify an obtuse angled triangle
    f. Identify a right angled triangle
2. You will also know that:
    a. The interior angles of a triangle add up to 180°.
    b. The exterior angle of a triangle equals the sum of the opposite angles
    c. Do calculations of unknown angles in any given triangle.

The triangle is very important to mathematics and the world around us. For example, in this unit you will see that the triangle is the basic building block of all two-dimensional shapes, including rectangles, squares, rhombuses, and kites.

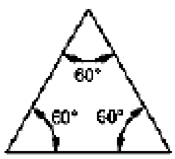
The buildings around us and the bridges we drive over are guaranteed to have triangles in their construction. The trigonometry behind triangles are used in technology including GPS systems, satellite imagery, and cell phone frequencies. As you work through this section, you'll begin to understand triangles and their value.[iii]

6.1 Triangle Classification

A triangle has three sides and three angles. The three interior angles always add up to 180°.

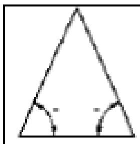
Equilateral, Isosceles and Scalene Triangles

There are three special names given to triangles that tell how many sides (or angles) are equal. These names must be **memorised**.

<table>
<tr>
<td>
</td>
<td>Equilateral Triangle

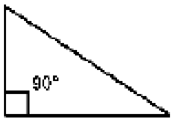
Three equal sides

Three equal angles, always 60°</td>
</tr>
</table>

	**Isosceles Triangle** Two equal sides Two equal angles
	**Scalene Triangle** No equal sides No equal angles

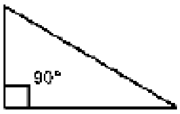
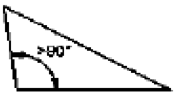
What Type of Angle?

Triangles can also have names that tell you what **type of angle** is inside:

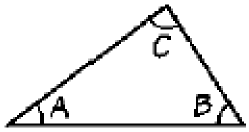
	**Acute Triangle** All angles are less than *90°*
	**Right Triangle** Has a right angle (*90°*)
	**Obtuse Triangle** Has an angle more than *90°*

What Type of Angle?

Triangles can also have names that tell you what **type of angle** is inside:

	**Acute Triangle** All angles are less than *90°*
	**Right Triangle** Has a right angle (*90°*)
	**Obtuse Triangle** Has an angle more than *90°*

6.2 Calculate Angles in Triangles



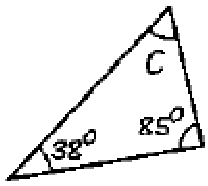
In a triangle, the three interior angles always add to 180°.:

$\angle A + \angle B + \angle C = 180°$

We can use that fact to find a missing angle in a triangle:

Example

Find the missing angle "C"



Start with: $\angle A + \angle B + \angle C = 180°$

Fill in what you know: $38° + 85° + \angle C = 180°$

Rearrange $\angle C = 180° - 38° - 85°$

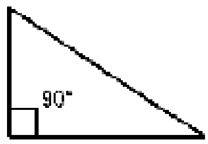
142

Calculate: $\angle C = 57°$

6.3 Right Triangles

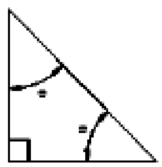
A **right-angled triangle** (also called a **right triangle**) is a triangle with a right angle (90°) in it.

The **little square** in the corner tells us it is a right-angled triangle.



The right-angled triangle is one of the most useful shapes in all of mathematics! It is used in the Pythagoras Theorem and Sine, Cosine and Tangent which you will learn about in the next unit.

There are two types of right-angled triangles:

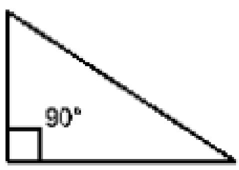


Isosceles right-angled triangle

One right angle

Two other **equal** angles always of **45°**

Two equal sides



Scalene right-angled triangle

One right angle

Two other **unequal** angles

No equal sides

	Go to your workbook and do exercise 6.1 and 6.2 as classwork or homework.

UNIT 6: PROPERTIES OF TRIANGLES

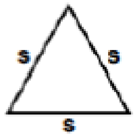
Exercise 6.1

Identify the types of triangles.

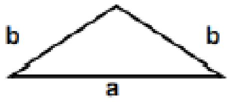
File all your work behind this page.

1.

2.



3.



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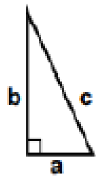
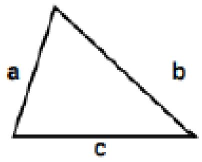
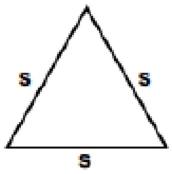


5.



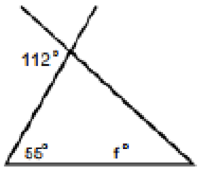
6.



Solve for the given variable.

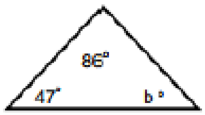
1.

$f\ =$



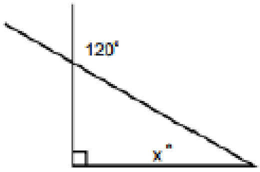
2.

$b\ =$



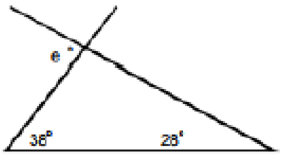
3.

$x\ =$



4.

$e\ =$

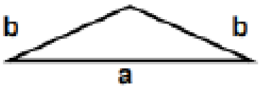


Exercise 6.2

Identify the types of triangles.

File all your work behind this page.

1.



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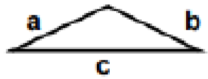

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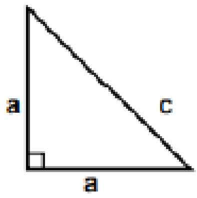

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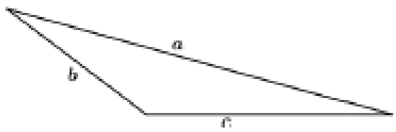

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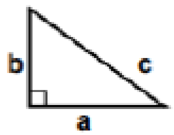

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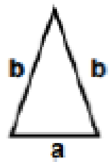
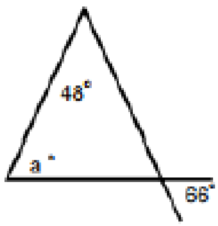
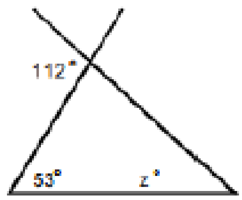


Solve for the unknown variable

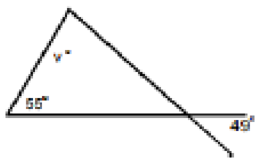
1. $a =$



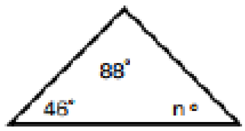
2. $z =$



3. $v =$

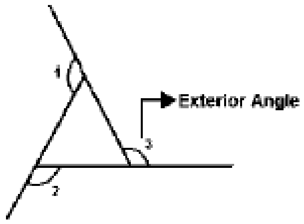


4. $n =$



6.4 The Exterior Angle of a Triangle is equal to the Sum of the Two Opposite Interior Angles

Any side of a triangle can be extended and then the angle that forms between that side and the triangle will be called the **exterior angle**.

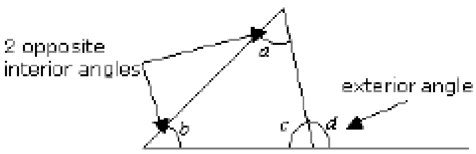


$\angle 1, \angle 2,$ as well as $\angle 3$ are all three exterior angles of the triangle.

In geometry there is a theorem that states that:

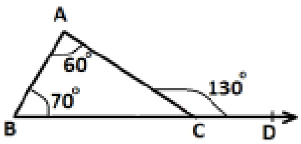
The exterior angle of a triangle is equal to the sum of the two opposite interior angles.

The following sketch shows what is meant by the terms "exterior angle" and "opposite interior angles":

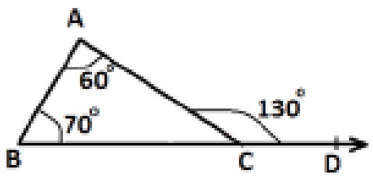


$d = a + b$

Example :



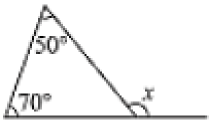
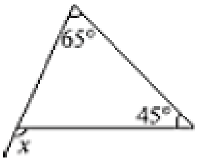
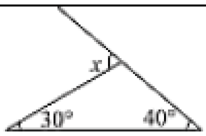
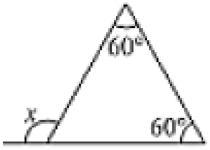
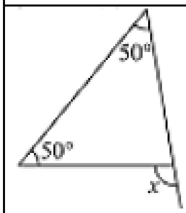
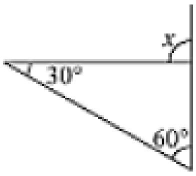
Example :



The reason why the exterior angle of the triangle is equal to the sum of the two opposite interior angles is this: if you subtract $60° + 70°$ from $180°$ you get $\angle C = 50°$. (The sum of interior angles of a triangle = $180°$.) If you subtract $\angle C = 50°$ from $180°$ you get $\angle ACD = 130°$. (Angles on a straight line = $180°$.) But $60° + 70° = 130°$, so the sum of the opposite interior angles is equal to the exterior angle.

	Go to your workbook and do exercise 6.3 as classwork or homework.

Exercise 6.3

	Calculate the unknown angle.
	1.
	2.
	3.
	4.
	5.
	6.

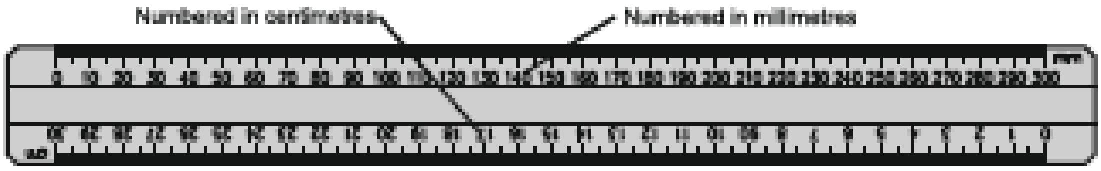
6.5 Draw and Construct Shapes using Mathematical Tools

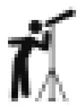
To be able to solve problems in geometry and trigonometry you need geometrical tools. For this course you will need a pencil (with a sharpener), a ruler, an eraser, a protractor and a compass.

6.5.1 Rulers

Most rulers used for drawing are made from plastic. More accurate rulers used for measuring in workshops are made from steel. We will only use simple rulers for this course.

Rulers come in different lengths mostly $15\ cm$ and $30\ cm$. They are divided into millimetres and the more accurate ones into half-millimetres. The numbers on the ruler are marked so that every 10th line represents either ten millimetres or one centimetre. Each single line represents one millimetre.





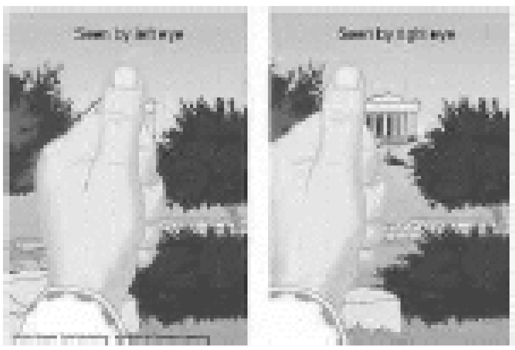
Parallax Error

Take note!

When you use a ruler or a protractor, you must make sure that your eyes are directly over the measurement you want to read. If you are a little bit to the left or to the right, you will get the wrong reading and your drawings will be incorrect.

To properly read or mark out using a ruler, the user's eyes must always be directly over the required point of the ruler. If either to the left or the right of the point, a larger or smaller value is read.

Here is an easy way to understand this. Look at something in front of you.



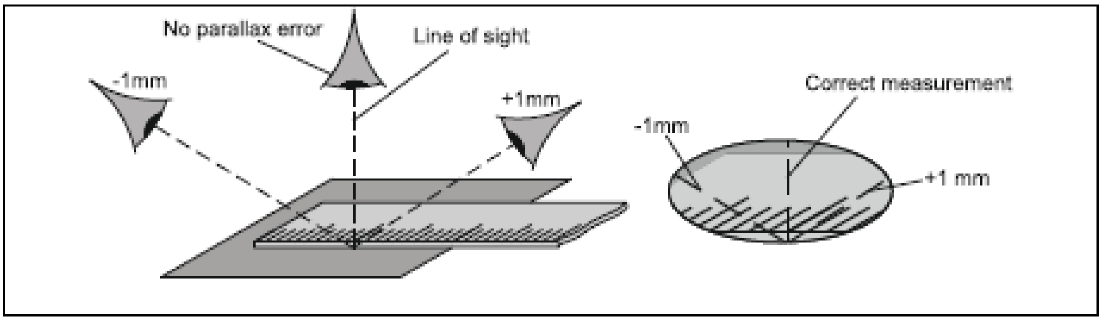
Hold up your thumb so that you can't see the object.

Now close your left eye and look at what you see

Then close your right eye and look at what you see.

The "picture" you see with one eye is different to what you see with the other eye.

With a ruler it would look like this:

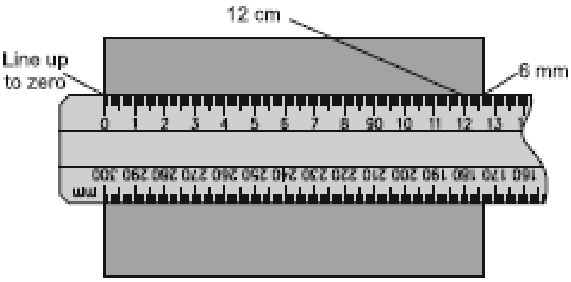


Using a Ruler

Accurate measurements in any mathematical drawing is very important. Measuring is quite easy, but you need to practice to get the right measurements from a ruler and transfer them to a drawing.

Example

Draw a line that is 126 mm long. 126 mm is 12 cm and 6 mm



Line the point '0' of the ruler up against what you want to measure or the point where you need to draw a line. In the example we used the end of the piece of paper.

Count the number of centimetre lines until you get to 12 cm.

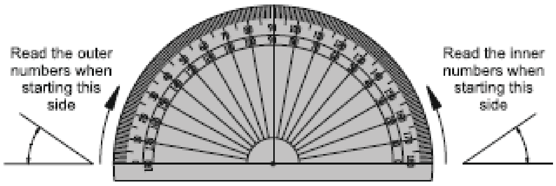
Then count out the 6 mm you need to complete the measurement.

Your measurement is now 12 cm which is 120 mm plus 6 mm making the total of 126 mm.

6.5.2 The Protractor

In geometry we use angles to solve problems. To draw angles we use a protractor. It is also used to measure an existing line on a drawing.

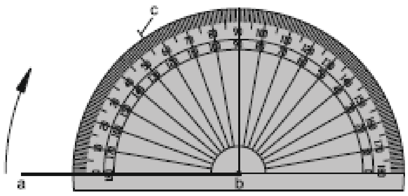
The protractor has two lines of numbers going in opposite directions. You must use the right line. When you are measuring on the left you use the numbers on the outside. When you are measuring on the right, you use the numbers on the inside.



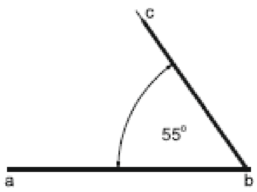
Example:

Using a protractor to draw an angle.

Draw the base line '*a-b*' to the right length with your ruler.



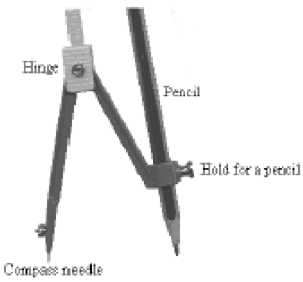
Put the protractor's centre at one end of the line '*b*'. Make sure the base line on the protractor is exactly on the line you drew. Make a small mark for the angle at the edge of the protractor '*c*'.



Join the point at the end of the line '*b*' to the mark '*c*'.

6.5.3 Construct Figures using a Ruler and Compass

A **compass** is an instrument used to draw circles or the parts of circles called arcs. It consists of two movable arms hinged together where one arm has a pointed end and the other arm holds a pencil.



To draw a circle (or arc) with a compass:

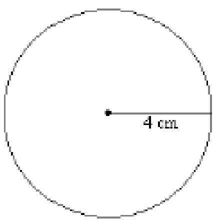
Make sure that the hinge at the top of the compass is tightened so that it does not slip

Tighten the hold for the pencil so it also does not slip

The pencil lead and the compass's needle must be even

Press down the needle and turn the knob at the top of the compass to draw a circle (or arc)

Example



Use a compass to draw a circle of radius 4 cm.

Use a ruler to set the distance from the point of the compass to the pencil's lead at 4 cm.

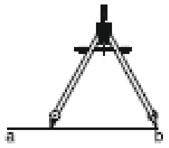
Place the point of the compass at the centre of the circle.

Draw the circle by turning the compass all the way around.[lv]

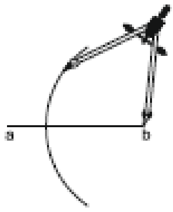
Bisect a Line into Two Equal Parts

Bisect means "to cut in half".

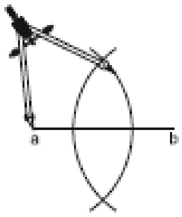
An arc is an incomplete circle.



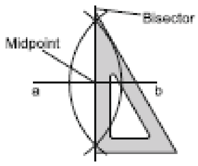
To bisect a line ($a - b$) use a compass, set the to a distance that is bigger than half the length of the given line.



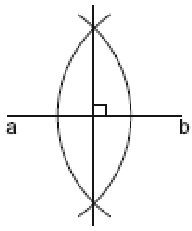
Put the point of the compass at one end of the line 'b' and draw an arc across the line.



Put the point of the compass at the other end of the line 'a' and draw another arc across the line.

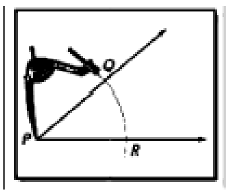


Use a set-square or ruler to draw a line to connect the points where the two arcs cross.

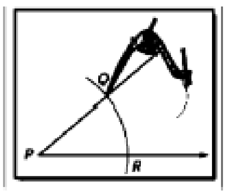


The new line is the **perpendicular bisector** and it is exactly in the middle of the line.

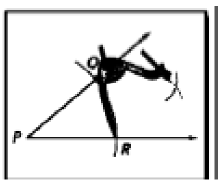
Bisect an Angle



Step One: Draw an angle. Put your compass at P and draw a large arc that intersects both sides of $\angle P°$. Label the points of the intersection Q and R.



Step Two: With the compass at point Q, draw and arc in the interior of the angle.



Step Three: Keeping the same compass setting, place the compass at point R and draw an arc that intersects the arc drawn in Step 2. Label the point of the intersection T.

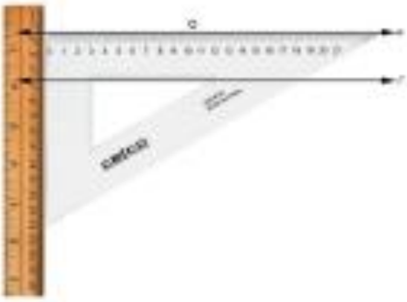
Drawing parallel lines

Draw a line. Use a triangle from your drawing set. Make a point (O) where the parallel line will be drawn.

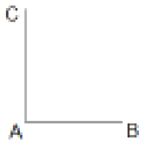
Place your ruler against the drawing triangle. Keep your ruler and triangle firmly in place.

Slide the triangle up to point O.

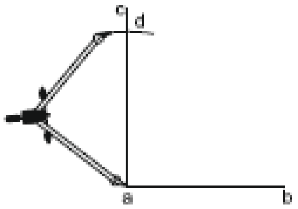
Draw a line through point O. You will then have a set of parallel lines.



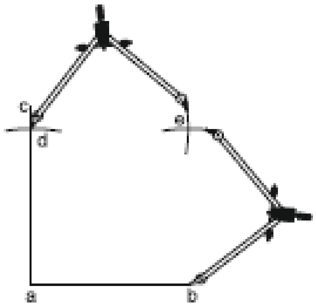
Drawing a perfect square



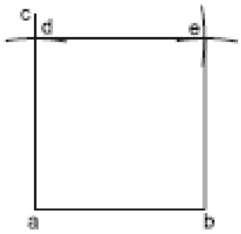
To the length of the '*A – B*', at '*A*', draw a perpendicular line *(90º) 'A-C'*.



Set the compass at the same length as '*a-b*'. Put the compass point on 'a' and draw an arc to cross the perpendicular line in '*d*'.



Put the compass point on point '*b*' and draw an arc, then set the compass point on point '*d*' and draw an arc to cross at '*e*'.



Draw a line from point '*d*' to point '*e*' and a line from point '*b*' to point '*e*' to complete the square.

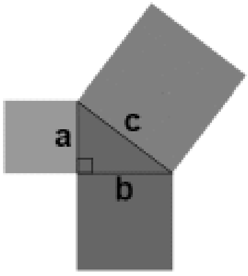
Go to your workbook and do exercise 6.4 as classwork or homework.

Exercise 6.4

Construct the following geometrical shapes. File all your work behind this page.
1. Draw a line that is $7cm$ long.
2. Draw a line that is $13mm$ long
3. Use your protractor to draw a $30°$ angle.
4. Use your protractor to draw a $120°$ angle.
5. Use you compass to draw a circle with a $5cm$ radius.
6. Use you compass to draw a circle with a diameter of $12cm$.
7. Use your ruler and your compass to bisect a line of $12cm$ into two equal parts.
8. Use your ruler and your compass to bisect an angle of $70°$ into two equal parts.
9. Use your ruler and your compass to draw a square with side length $10cm$.
10. Draw a line of your choice. Draw a line parallel to the first line, $5cm$ apart from the first one.

6.6 Pythagoras' Theorem

Over 2000 years ago there was an amazing discovery about triangles:



When a triangle has a right angle (90°) and squares are made on each of the three sides, then the biggest square has the **exact same area** as the other two squares put together!

It is called "Pythagoras' Theorem" and can be written in one short equation: $a^2 + b^2 = c^2$.

Note: c is the **longest side** of the triangle and a and b are the other two sides

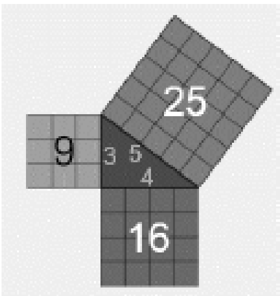
The longest side of the triangle is called the "hypotenuse" (meaning "stretched"), so the **formal definition** is:

In a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Example:

A "Pythagorean Triple" is a set of positive integers, a, b and c that fits the rule:

$a^2 + b^2 = c^2$. The smallest Pythagorean Triple is $3, 4$ and 5.



This triangle has a right angle in it. The sides are $3, 4$ and 5.

Let's check if the areas are the same:

$3^2 + 4^2 = 5^2$

Calculating this becomes:

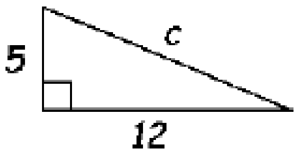
$9 + 16 = 25$

Why is this useful? Let's look at some examples:

If we know the lengths of two sides of a right-angled triangle, we can find the length of the third side. (But remember it only works on right angled triangles!)

Example 1:

Start with: $a^2 + b^2 = c^2$.



Put in what we know: $5^2 + 12^2 = c^2$

Calculate squares: $25 + 144 = c^2$

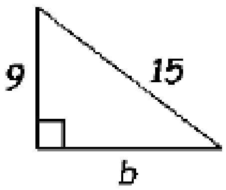
$c^2 = 169$

Now find the square root of 169; $\sqrt{169} = 13$

So now we know that $c = 13$

Example 2:

Start with: $a^2 + b^2 = c^2$.



Put in what we know: $9^2 + b^2 = 15^2$

Calculate squares: $81 + b^2 = 225$

Solve for b^2.

Calculate: $b^2 = 225 - 81$

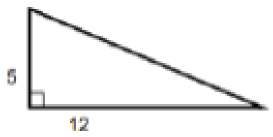
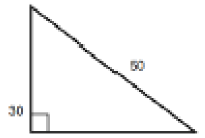
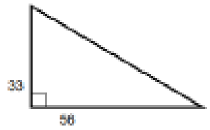
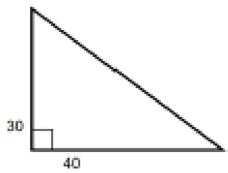
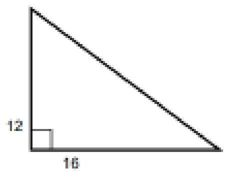
$= 144$

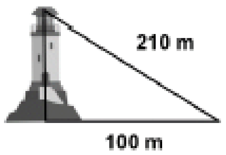
$b = \sqrt{144}$

$= 12$

☺	Go to your workbook and do exercise 6.5 as classwork or homework.

Exercise 6.5

	Find the length of the third side of each triangle. (Pythagoras Theorem)
1. 	2. 
3. 	4. 
5. 	6. 
7. 	8. 

9 Use the sketch to calculate the height of the lighthouse.	
10. How far away is the boy standing from the tree?